Distributed Algorithms for Internet-of-Things-enabled Prosumer Markets: A Control Theoretic Perspective*

**Syed Eqbal Alam · Robert Shorten ·
Fabian Wirth · Jia Yuan Yu**

Abstract *Internet-of-Things* (IoT) enables the development of sharing economy applications. In many sharing economy scenarios, agents both produce as well as consume a resource; we call them *prosumers*. A community of prosumers agrees to sell excess resource to another community in a prosumer market. In this chapter, we propose a control theoretic approach to regulate the number of prosumers in a prosumer community, where each prosumer has a cost function that is coupled through its time-averaged production and consumption of the resource. Furthermore, each prosumer runs its distributed algorithm and takes only binary decisions in a probabilistic way, whether to produce one unit of the resource or not and to consume one unit of the resource or not. In the proposed approach, prosumers do not explicitly exchange information with each other due to privacy reasons, but little exchange of information is required for feedback signals, broadcast by a central agency. In the proposed approach, prosumers achieve the optimal values asymptotically. Furthermore, the proposed approach is suitable to implement in an IoT context with minimal demands on infrastructure. We describe two use cases; community-based

Syed Eqbal Alam · Jia Yuan Yu
Concordia Institute for Information Systems Engineering, Concordia University, Montreal, Quebec, Canada

Robert Shorten
School of Electrical, Electronic and Communications Engineering, University College Dublin, Dublin, Ireland

Fabian Wirth
Faculty of Computer Science and Mathematics, University of Passau, Passau, Germany

car sharing and collaborative energy storage for prosumer markets. We also present simulation results to check the efficacy of the algorithms.

Keywords Distributed optimization · Internet-of-Things (IoT) · Optimal control · Optimal allocation · Prosumers · Sharing economy · Prosumer markets.

1 Introduction and Setting

Recently, consumers across a range of sectors have started to embrace shared ownership of resources and services with guaranteed access, as opposed to more traditional business models that focus on sole-ownership only. The reasons for this trend are multi-faceted and range from societal issues, such as the need to reduce wastage, and more general environmental concerns [1–4], to pure monetary opportunities arising from increased connectivity (and the ability that this gives to advertise the availability of unused resources and services) [5]. Well-known examples of successful companies building *sharing economy* products include Airbnb (hospitality), Lyft (ride sharing) [6], Bird and Lime (scooter sharing), Mobike (bike sharing) [3], and Google (Google reviews—information sharing).

Roughly speaking, several types of sharing application classes are discerned (as described in [5]).

A. **Opportunistic sharing:** Services based on opportunistic sharing of resources exploit large-scale availability of either unused resources or obsolete business models or both. Examples of products in this area include the parking application JustPark (`www.justpark.com`) and the peer-to-peer car sharing services Getaround (`www.getaround.com`). The key enablers for such products are mechanisms for informing agents of available resources, their delivery, and payments.

B. **Federated negotiation and sharing:** Here, groups of agents come together to negotiate better contracts with utilities (electricity, gas, water, health), or to provide mutually beneficial services such as collaborative storage of energy. The key enablers for such products are mechanisms for grouping communities and for enforcing contractual obligations for federations of like-minded consumers.

C. **Bespoke sharing:** In this case, products are designed with the specific objective of being shared, rather than for sole ownership. A basic example of such systems is devices and services that allow sharing of a single electric charge-point by several users. Other examples include time-shared apartments or cars that are owned by several people rather than a single person [5].

D. **Hybrid sharing:** Finally, opportunities also exist for sharing economy to support the regular economy. We readily find examples of such systems in the hospitality industry, where ad-hoc sharing economy infrastructure (spare rooms in local houses) can be used as a buffer to accommodate excess demand in the regular economy (hotels).

The common characteristic in all of the above application classes is the ability for community-wide communication and actuation, both to enable services to be bought and sold, and so that contracts can be enforced.

The development of sharing economy applications [7, 8] is facilitated by Internet-of-Things (IoT). For example, IoT helps to perform secure payments, to track the location and the condition of an object, to list a few. Interested readers can find several IoT-based applications in [9, 10] and the papers cited therein. Therefore, while the value of the sharing economy is not in question [2, 11] and while many of the essential infrastructural elements needed for the deployment of such systems are being developed rapidly, there is an additional requirement for structured platforms to enable distributed *community-wide* buying and distributed *community-wide* selling. Currently, such platforms are at a very early stage of development with significant opportunities for improvement.

Our objective in this chapter is to address this deficit partially and to develop tools to support the design of community-based prosumer markets. We define *prosumers* as agents that both produce and consume a resource [12]. Specifically, we are interested in developing *light* algorithms that can easily be deployed on modest IoT platforms, and that can be used to support distributed community-wide buying and selling of resources. Here, by *light*, we mean algorithms that place low demands on the infrastructure, both in terms of computational power, and actuation and connectivity requirements of individual prosumers. A fundamental requirement is also that such algorithms are scale-free in the sense that they can operate across a range of community sizes; from small communities of a few prosumers to larger communities made up of very many prosumers. These constraints are directly related to the challenges associated with uncertainties that arise in the context of sharing economy problems. Typically, at any time instant, one does not know how many prosumers are participating in the sharing scheme; whether prosumers can or are willing to communicate with each other (perhaps due to privacy considerations), and whether enough computational power is available to the whole network to allocate resources in real-time optimally. A further complication is that we would like any scheme that we develop to be backward compatible with old IoT platforms that support only essential interaction between prosumers and infrastructure. Thus, there is considerable interest in developing *light* algorithms that place only modest demands on infrastructure, yet can be used to implement complex policies in the face of the uncertainties mentioned

above.

Given this context, we are particularly interested in situations where communities come together to purchase and sell related commodities simultaneously. Such systems arise, for example, in energy systems where agents (prosumers) both produce and consume energy [12–14].

2 Prosumer Markets and Communities

Prosumers are the agents that both produce and consume resources [12, 15]. We are interested in prosumer markets that facilitate a community of prosumers for distributed production and consumption. Such markets are emerging rapidly in energy sector [16, 17], but also in other areas such as shared mobility [18]. Parag and Sovacool [19] classify prosumer markets according to three network architectures [1].

(i) *Prosumer-to-prosumer model:* In this peer-to-peer model, prosumers interact (buy or sell resource) directly with each other as depicted in Figure 1. This model is widespread. For example, consider the case of car sharing platform Turo. Here, car owners list their cars on Turo sharing platform, and the riders book the cars of their choice through this platform for a certain period with a fee. A similar, peer-to-peer model is proposed in [20] for energy trading in microgrids.
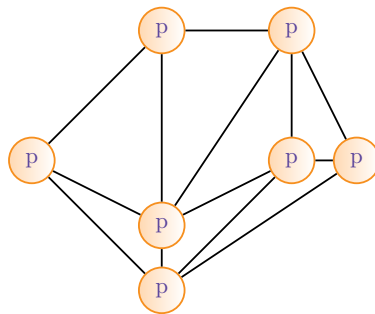

Fig. 1: Prosumer-to-prosumer model. Here p represents a prosumer. Figure adapted from [19].

(ii) *Prosumer-to-firm model:* In this model, prosumers interact with a local firm directly. There are two types of prosumer to firm models, *prosumer-to-interconnected-firm* model, and *prosumer-to-isolated-firm* model. In the

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[1] In the network architectures, p represents a prosumer.

prosumer-to-interconnected-firm model, prosumers are connected to a local firm, which may be connected to the main firm as presented in Figure 2(a). For example, suppose that prosumers produce energy from renewable sources and are connected to a microgrid. A prosumer satisfies its energy needs from the microgrid and the energy it produces. If the prosumer produces more energy than it needs; then, it can return excess energy to the microgrid, this microgrid may be connected to the main grid. Whereas, in the prosumer-to-isolated-firm model, prosumers are connected to the local firm, which works in isolation as depicted in Figure 2(b). Example of the prosumer-to-isolated-firm model is Island microgrid [21] in which prosumers and microgrid work together to fulfill the energy need of prosumers in the Island.
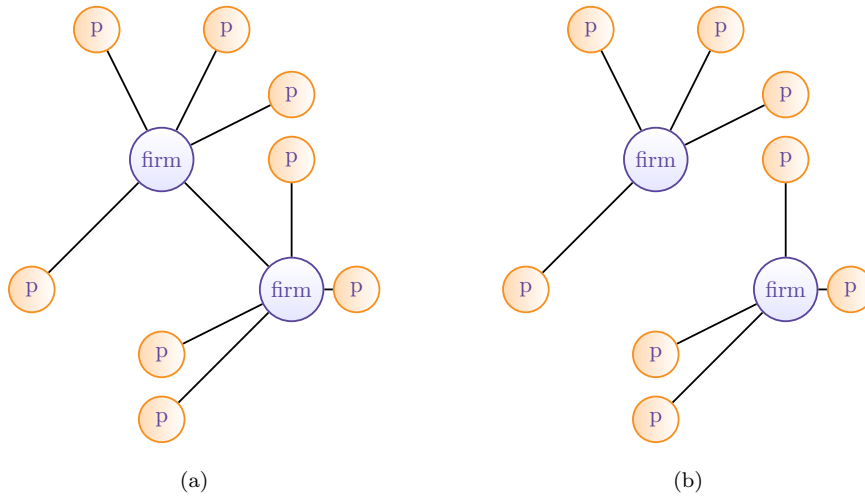

Fig. 2: Prosumer-to-firm models—(a) prosumer-to-interconnected-firm model, (b) prosumer-to-isolated-firm model. Figure adapted from [19].

(iii) *Community-based prosumer model:* In this model, prosumers are located in the same geographic location who have similar resource needs and resource production pattern; more generally, they share common goals and interests. These prosumers are grouped to interact with each other and efficiently manage the resource needs of the community, as depicted in Figure 3. In this case, communities may also exchange resources with each other. A recent example of community-based trip sharing is found at [22] in which the algorithm clusters commuters in communities to optimize car usage. We clarify that, for simplicity, we consider a single prosumer community

that interacts with another community (*external community*) in the rest of the chapter unless otherwise stated.
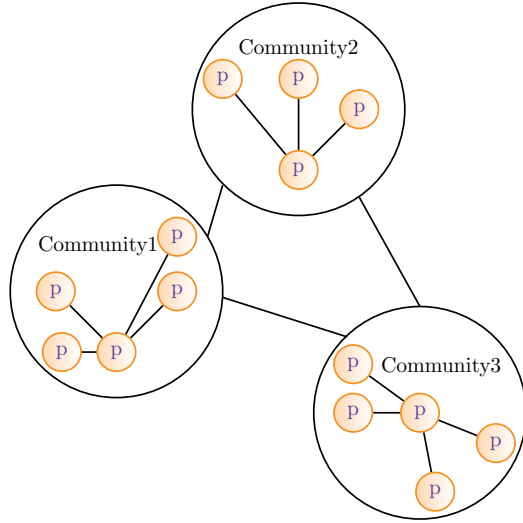

Fig. 3: Community-based prosumer model. Figure adapted from [19].

While work on analytics to help design prosumer markets is still in its infancy, somewhat surprisingly few papers have begun to deal with some of the complex market design issues associated with such systems [13], [23], [24], [25]. Roughly speaking, these papers deal with two main issues; (i) the existence of market equilibria, and (ii) methods to allocate resources amongst (competing) prosumers. It is in this latter context that this present work is placed. Generally speaking, resource allocation algorithms for prosumer markets have until now been formulated in an optimization context and can be categorized as is traditionally done for classical optimization models. Namely, resources are either allocated centrally [26] as in the case of Uber, whereby drivers are assigned to the passengers; or in a distributed fashion as in the case of Airbnb, whereby guests and hosts choose each other; or using hybrid of the above two, as in the case of Didi Chuxing [1]. As we have said, typically, these allocation problems are formulated in an optimization context, paying particular attention to the certain constraints that arise in the sharing economy. These include privacy of the individual, fair allocation of resources, the satisfaction of service level agreements, and ever-increasing regulatory constraints (for example, in the case of Airbnb).

Our contribution in this chapter is to address these problems using a different approach. Namely, we shall consider these problems (in a control theoretic

context) as regulation problems with optimality constraints. In particular, we are interested in applications where prosumer communities both buy and sell resources from or to one, or more external entities. Thus, we take the view that such communities have a contract to both buy and sell a pre-specified level of a resource at a given time instant, and the objective of the allocation algorithm is to ensure that these levels of demand are met. Given this basic setting, we ask the question as to whether this can be done without any explicit exchange of information between individual prosumers in situations where prosumers take only binary decisions (buy or sell), and whether given these constraints, optimal use of the resource can be realized. We shall see in the next section that it is indeed possible to formulate the problem and develop distributed algorithms that achieve all of these properties, and which can be implemented in an IoT context with minimal demands on infrastructure.

2.1 Prior Work

Recent analytics work on sharing economy has primarily followed two directions; (i) market design and equilibria, and (ii) optimal allocation of resources. Here, we give a very brief picture of some of the most recent work. In [25], Georgiadis et al. propose three types of resource allocation mechanisms; centralized, coalition-based (game-theoretic approach), and peer-to-peer. In [23], Gkatzikis et al. propose a collaborative consumption mechanism to minimize the electricity cost of a community, and in [27], Tushar et al. developed a game-theoretic model for peer-to-peer energy trading. Furthermore, in [13], Moret et al. design a community-based distributed energy collective market model that helps energy prosumers to optimize their energy resources and to achieve social welfare of the community. More recently, Iosifidis and Tassiulas [24], propose optimization techniques for resource exchange and production scheduling for cooperative systems. Courcoubetis and Weber [28], propose mechanisms for the optimal allocation of shared computing resources. In [22], Hasan et al. propose a community-based car sharing model to maximize the trip sharing. To do so, they use mixed-integer programming, graph theory, and clustering techniques. In [20], Zhang et al. propose a game theoretic peer-to-peer energy trading model between local prosumers and distributed energy resources. They show that the model gives rise to an equilibrium between energy production and consumption. In addition to these works, a peer-to-peer equilibrium model for collaborative consumption is proposed by Benjaafar et al. in [29]. Finally, Grijalva et al. [18], discuss architecture for prosumer-based distributed control for the electricity grid. We refer interested readers to a recent review paper by Sousa et al., which covers market design, optimization techniques, and interesting future directions at [30].

2.2 Contribution

We are primarily interested in applications where prosumer communities buy and sell resources via contracts to one or more external entities and must ensure that they meet certain demands in real-time. Thus, the objective of any allocation algorithm is to ensure that these levels of demand are met. Typically, a problem of this nature would be solved in a standard optimization framework. Unfortunately, this approach is not available to us due to the uncertainty that prevails for this system class. For example, the number of prosumers participating at a given time instant may vary, as may the contracted level of prosumption. Also, for reasons of privacy, prosumers may only communicate with each other or with the infrastructure in a limited fashion, making the communication graph unknown *a-priori*, from the point of view of algorithm development. Thus, our approach is to formulate these allocation problems in a control theoretic setting, where the effect of these uncertainties and other disturbances can be dealt with using feedback. Our principal contribution is, therefore, to develop distributed control algorithms that can asymptotically achieve optimality.

3 Problem Statement

Let us assume that a prosumer market consists of a prosumer community and an *external community*. The prosumer community has $N \in \mathbb{N}$ prosumers producing and consuming a resource, which agrees to sell its excess resource to the external community. We also assume that the prosumer market has a control unit, which measures the aggregate consumption and production of the resource. It communicates with the prosumer community as well as the external community; we call it a *sharing platform*. Additionally, notice that N is not known to the individual prosumers participating in the scheme. For simplicity, we assume that there is a single resource produced and consumed by all prosumers, though our formulation can easily be extended to multiple resources and multiple prosumer communities. In our model, the process of production and consumption takes place at discrete time instants $t_0 < t_1 < t_2 < \ldots$, where $t_0 = 0$. At each time instant, the overall production and consumption of the previous time instant are evaluated and adjusted. Additionally, we assume that communities and external agencies are contracted to, *on aggregate*, consume and produce a certain amount of the resource at time instant t_k, for $k = 0, 1, 2, \ldots$

We assume that each prosumer has limited actuation and at any time instant t_k, for $k = 0, 1, 2, \ldots$, either it consumes one unit of the resource or it does not consume it. Similarly, each prosumer either produces one unit of the resource at a time instant or it does not produce it. Thus, for each prosumer i, we denote by $x_i(k)$ the amount of the resource consumed, and by $y_i(k)$ the

amount of the resource produced at time instant t_k.[2]

We also assume that there are constants $C_x \geq 0$ and $C_y \geq 0$, specifying the aggregate consumption and production bounds. Notice that the constants C_x and C_y are known to the sharing platform, but not to individual prosumers in the market. Thus, at each time instant t_k, we require:

$$\sum_{i=1}^{N} x_i(k) = C_x, \quad \text{and,} \tag{1}$$

$$\sum_{i=1}^{N} y_i(k) = C_y. \tag{2}$$

Our primary objective is to ensure that these prosumption bounds are met at each time instant t_k, for $k = 0, 1, 2, \ldots$ However, we are particularly interested in situations where production and consumption are coupled together. For example, in communities that are formed to produce energy, the time-averaged production and consumption of energy might be coupled through battery storage requirements. Furthermore, in the community-based car sharing prosumer market, the average number of delivered and received "cars" might be coupled through the desired value of utilization of cars; and similarly, production of a resource might depend on its consumption. Thus, in these situations, we would like to ensure that consumption and production bounds (1) and (2) are met asymptotically. To formulate this as a long-term requirement, we introduce the time-averaged consumption:

$$\overline{x}_i(k) \triangleq \frac{1}{k+1} \sum_{\ell=0}^{k} x_i(\ell), \quad \text{for } i = 1, \ldots, N, \tag{3}$$

with the time-averaged production $\overline{y}_i(k)$ defined analogously. Now, let $T_i \in \mathbb{R}_+$ be the desired value of utilization of the resource, for $i = 1, \ldots, N$. Thus, depending on the application (refer Section 5, cf. (25)), we might require:

$$\lim_{k \to \infty} \overline{x}_i(k) + \overline{y}_i(k) = T_i, \quad \text{for } i = 1, \ldots, N, \tag{4}$$

with some additional constraints on $\overline{x}_i(k)$ and $\overline{y}_i(k)$. In several applications, we might require for $\alpha_i \in [0, 1]$:

$$\lim_{k \to \infty} \overline{x}_i(k) = \alpha_i T_i, \quad \text{and,} \lim_{k \to \infty} \overline{y}_i(k) = (1 - \alpha_i)T_i, \quad \text{for } i = 1, \ldots, N. \tag{5}$$

Fortunately, in problems that we consider, there is flexibility in satisfying these constraints and it is enough to satisfy that for sufficiently large k, we have:

$$\overline{x}_i(k) + \overline{y}_i(k) \approx T_i, \quad \text{and,} \tag{6}$$

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[2] Depending on the application, the processes of production and consumption may be more appropriately modeled on a continuous time-scale. In this case, we interpret time instants t_k at those times in which the prosumption over the interval $(t_{k-1}, t_k]$ is accounted for.

$$\overline{x}_i(k) \approx \alpha_i T_i, \quad \text{and,} \quad \overline{y}_i(k) \approx (1 - \alpha_i)T_i, \qquad (7)$$

for $i = 1, \ldots, N$. To formulate this mathematically, we associate a cost $g_i : (0, 1]^2 \to \mathbb{R}_+, (x_i, y_i) \mapsto g_i(x_i, y_i)$ to the deviation of the actual long-term prosumptions.

Assumption 1 (Cost function) *The cost function $g_i(\cdot)$ is strictly convex, strictly increasing in each variable, and is continuously differentiable, for all i.*

Given this basic setting, we are interested in solving the following optimization problem:

Problem 1 (Optimization)

$$\min_{\xi_1, \ldots, \xi_N, \eta_1, \ldots, \eta_N} \quad \sum_{i=1}^{N} g_i(\xi_i, \eta_i), \qquad (8)$$

$$\text{subject to} \quad \sum_{i=1}^{N} \xi_i = C_x, \qquad (9)$$

$$\sum_{i=1}^{N} \eta_i = C_y, \qquad (10)$$

$$\xi_i \geq 0, \qquad (11)$$

$$\eta_i \geq 0, \quad \text{for } i = 1, 2, \ldots, N. \qquad (12)$$

We assume that for this optimization problem an optimal point $(\xi^*, \eta^*) \in \mathbb{R}_+^{2N}$ exists. The optimal point is unique by Assumption 1 of strict convexity of the cost function $g_i(\cdot)$. As our problem is timed, we aim to design a distributed algorithm determining, for each time instant t_k, values of $x_i(k)$ and $y_i(k)$, such that for the long-term averages, we have:

$$\lim_{k \to \infty} \overline{x}_i(k) = \xi_i^*, \quad \lim_{k \to \infty} \overline{y}_i(k) = \eta_i^*, \quad \text{for } i = 1, \ldots, N. \qquad (13)$$

Also, it is desirable that the constraints of the optimization problem are satisfied by $\overline{x}_i(k), \overline{y}_i(k)$, for every k, where $\overline{x}_i(k)$ takes the role of the optimization variable ξ_i and $\overline{y}_i(k)$ that of η_i.

Typically, a problem of this nature can be solved in a standard optimization framework. Unfortunately, this approach is not available to us for many reasons:

(i) The number of prosumers N in the prosumer community, and the constraints C_x and C_y may vary with time, making an offline computation of an optimal solution difficult.

(ii) To preserve privacy, we also assume that prosumers do not necessarily communicate with each other, and they only communicate with the infrastructure in a limited fashion. Thus, even the communication graph is unknown *a-priori* for this particular problem class. In particular, individual cost function g_i is often private and is not shared by prosumers.

(iii) We are interested in algorithms that self organize and converge to an optimal solution even in the presence of disturbances in the state information.

Our approach, therefore, is to treat the above problem as a feedback (stochastic) control problem, which changes the formulation in a minor way; namely, we allow:

$$\sum_{i=1}^{N} x_i(k) \approx C_x, \text{ and } \sum_{i=1}^{N} y_i(k) \approx C_y, \quad \text{for all } k. \tag{14}$$

In other words, we allow the instantaneous prosumption to undershoot or overshoot the reference values by a small amount. Then, given this background, and from Assumption 1 for the cost function g_i, for all prosumers i, we shall demonstrate that an elementary feedback control algorithm can be devised to solve an approximate version of Problem 1. As we shall see, this algorithm requires only a few bits of message transfer as intermittent feedback from a control unit (sharing platform) to prosumers in the prosumer market, but no inter-prosumer communication is required.

4 Algorithms for Community-based Prosumer Markets

The algorithms proposed in this section are motivated by the following elementary argument. We consider only the case in which at time instant t_k, either 0 or 1 unit of the resource is consumed or produced, for $k = 0, 1, 2, \ldots$ For the sake of argument, we only consider the case of pure consumption in this preamble.

Let $z(k)$ denote the number of times an agent (consumer) consumes a unit resource until time instant t_k, where $k = 0, 1, 2, \ldots$ Let $\overline{z}(k) \triangleq \frac{1}{k+1} z(k)$, denote the time-averaged consumption of the resource until time instant t_k. We assume that the consumption at time instant t_k is decided by a stochastic procedure, where the probability of consumption of one unit of the resource is a function $p(\overline{z}(k))$. Additionally, we assume that this probability conditioned on $\overline{z}(k)$ is independent of the previous history of the process. Then:

$$z(k+1) = z(k) + w(k), \tag{15}$$

where $w(k)$ is a random variable taking the value 0 or 1. Thus, the following holds true:

$$\overline{z}(k+1) = \frac{k+1}{k+2}\overline{z}(k) + \frac{1}{k+2}w(k) \qquad (16)$$

$$= \overline{z}(k) + \frac{1}{k+2}\big(w(k) - \overline{z}(k)\big). \qquad (17)$$

Recall that $p(\overline{z}(k))$ denotes the probability that $w(k) = 1$ at time instant t_k, for $k = 0, 1, 2, \ldots$ Then, we rewrite (17) as:

$$\overline{z}(k+1) = \overline{z}(k) + \frac{1}{k+2}\big(p(\overline{z}(k)) - \overline{z}(k)\big) + \frac{1}{k+2}\big(w(k) - p(\overline{z}(k))\big).$$

Note that systems of this form are discussed extensively in [31]. In particular, the term $\frac{1}{k+2}\big(w(k) - p(\overline{z}(k))\big)$ is a martingale difference sequence and is treated as noise. It is shown in [31] that under mild assumptions, $\overline{z}(k)$ converges almost surely. The basic idea of the remainder of this section is to construct stochastic feedback algorithms that mimic this argument. In particular, our basic idea in the sequel is to choose the probability distribution $p(\overline{z}(k))$, so that the stochastic system both solves a regulation problem and also optimization problem of the form of Problem 1, simultaneously.

4.1 Optimality Conditions

In this subsection, we briefly discuss the optimality conditions for Problem 1, using Lagrangian multipliers. These optimality conditions lead to the state dependent probabilities that we alluded to in the preamble.

Let $x = (x_1, x_2, \ldots, x_N)$ and $y = (y_1, y_2, \ldots, y_N)$. Also, let μ^1, μ^2 and $\lambda^1 = (\lambda^1_1, \ldots, \lambda^1_N)$, $\lambda^2 = (\lambda^2_1, \ldots, \lambda^2_N)$ be the Lagrange multipliers corresponding to the equality constraints (9), (10) and the inequality constraints (11), (12), respectively. The Lagrangian of Problem 1 is defined as $\mathcal{L} : \mathbb{R}^{2N} \times \mathbb{R}^2 \times \mathbb{R}^{2N} \to \mathbb{R}$, where

$$\mathcal{L}(x, y, \mu^1, \mu^2, \lambda^1, \lambda^2) = \sum_{i=1}^{N} g_i(x_i, y_i) - \mu^1\Big(\sum_{i=1}^{N} x_i - C_x\Big) - \mu^2\Big(\sum_{i=1}^{N} y_i - C_y\Big)$$

$$+ \sum_{i=1}^{N} \lambda^1_i x_i + \sum_{i=1}^{N} \lambda^2_i y_i.$$

We assume that the optimal value of Problem 1 is obtained for positive values of consumption and production. We, therefore, let $x^*_i, y^*_i \in (0, 1]$ denote the optimal point of Problem 1. By this assumption, the inequality constraints are not active, and it follows that the corresponding optimal Lagrange multipliers are $\lambda^{*1} = 0 = \lambda^{*2}$.

Additionally, let μ^{*1}, μ^{*2} be the optimal Lagrange multipliers for the equality constraints. The first order optimality condition is that the gradient vanishes, and inspection shows that the gradient condition decouples. Recall that $\nabla_x g_i(.)$ denotes the partial derivative of $g_i(.)$ with respect to x_i and $\nabla_y g_i(.)$ denotes the partial derivative of $g_i(.)$ with respect y_i. Then, we arrive at the following conditions:

$$\nabla_x g_i(x_i^*, y_i^*) = \mu^{*1}, \quad \text{for } i = 1, 2, \ldots, N,$$

and,

$$\nabla_y g_i(x_i^*, y_i^*) = \mu^{*2}, \quad \text{for } i = 1, 2, \ldots, N.$$

In other words, we have:

$$\nabla_x g_i(x_i^*, y_i^*) = \nabla_x g_j(x_j^*, y_j^*), \text{ for } i, j \in \{1, 2, \ldots, N\}. \tag{18}$$

The same consensus condition holds for $\nabla_y g_i(x_i^*, y_i^*)$, $i = 1, \ldots, N$. We find that the optimal values satisfy all the Karush-Kuhn-Tucker (KKT) conditions, which are necessary and sufficient conditions for optimality of differentiable convex functions (Chap. 5.5.3 [32]). Hence, the derivatives of the cost functions of all prosumers with respect to consumption as well as production must reach consensus at the optimal point.

This type of consensus condition has been used in [33, 34] (single resource case) and [35] (multi-resource case) to derive place-dependent probabilities that ensure convergence to the consensus condition and thus, to the optimal point.

4.2 Algorithm for Consumption

Here, we briefly describe the distributed algorithm proposed in [34] for allocating a single resource to consumers. In this model, no production is taking place, hence the corresponding variables are omitted. Suppose that there are N consumers in a community. We assume that consumer i of the community has a cost function $g_i : (0, 1] \to \mathbb{R}_+$, which is strictly convex, strictly increasing in each variable, and continuously differentiable, for $i = 1, 2, \ldots, N$. The random variable $x_i(k) \in \{0, 1\}$ denotes the consumption of the unit resource for consumer i at time instant t_k, for all i and k. As before, let $\overline{x}_i(k)$ be the time-averaged consumption of consumer i until time instant t_k, for all i and k.

The idea is to choose probabilities so as to ensure convergence to the social optimum and to adjust overall consumption by the community to the reference value (capacity) C_x, by applying a *feedback signal* $\Omega(k)$ to the probabilities. At each time instant t_k, the control unit updates $\Omega(k)$ using a gain parameter $\tau > 0$, the past aggregate consumption of the resource, and the capacity C_x

as described in (19) and then, broadcasts the new value to all consumers in the community:

$$\Omega(k+1) \triangleq \Omega(k) - \tau\Big(\sum_{i=1}^{N} x_i(k) - C_x\Big). \tag{19}$$

After receiving this signal, consumer i responds in a probabilistic way. The probability distribution $\sigma_i(\cdot)$ is calculated using the time-averaged consumption $\overline{x}_i(k)$ and the derivative $g_i'(\cdot)$ of the cost function $g_i(\cdot)$, as follows:

$$\sigma_i(\Omega(k), \overline{x}_i(k)) \triangleq \Omega(k)\frac{\overline{x}_i(k)}{g_i'(\overline{x}_i(k))}. \tag{20}$$

Notice that the cost function g_i is chosen as increasing function in each variable so that the probability $\sigma_i(\cdot)$ is in the valid range, for all i. Now, consumer i updates its resource consumption at each time instant t_k, either by consuming one unit of the resource or not consuming it, as follows:

$$x_i(k+1) = \begin{cases} 1 & \text{with probability } \sigma_i(\Omega(k), \overline{x}_i(k)); \\ 0 & \text{with probability } 1 - \sigma_i(\Omega(k), \overline{x}_i(k)). \end{cases}$$

Empirical results show that the time-averaged consumption $\overline{x}_i(k)$ converges to the optimal value x_i^* asymptotically.

Remark 1 (Integral control action) Equation (19) defines what is called an integral control action in tracking control. The overall consumption by the community should "track" the available capacity, i.e., approximate it. In other words, the objective of the integrator is to ensure that the tracking error $e(k) = \sum_{i=1}^{N} x_i(k) - C_x \approx 0$ asymptotically.

Remark 2 (Consensus of derivatives) The policy for $\sigma_i(\Omega(k), \overline{x}_i(k))$ in (20) is to ensure that asymptotically, $g_i'(\overline{x}_i(k)) = g_j'(\overline{x}_j(k))$, for all consumers i and j. As is discussed in [34], the convergence of the above algorithm and strict convexity of all cost functions (and of course assuming the existence of a feasible solution in the constraint set) is enough to imply that Problem 1 is solved asymptotically for the consumption case.

4.3 Algorithm for Coupled Prosumption

The case of *coupled prosumption* of a single resource is characterized by the constraint (4), that is, consumption and production are coupled through the desired value of utilization of the resource. The algorithm follows the case of exclusive consumption closely taking into account the cost function as discussed in the problem formulation (Section 3). Before proceeding, note that the following discussion extends to an arbitrary number of resources, but is presented for a single resource, both to aid exposition and to be consistent with the application class that is the principal consideration in this chapter.

Interested readers can look at [36] for preliminary results on multi-resource allocation. With this background in mind, following the discussion for consumption, let us assume that in a prosumer market there is a community of N prosumers. The prosumer community sells the excess resource to an external community. Furthermore, let τ_x, τ_y denote the gain parameters for consumption and production, $\Omega_x(k)$, $\Omega_y(k)$ denote the *feedback signals* for both processes, and C_x, C_y represent the respective contract (capacity) constraints. We assume the existence of a centralized control unit (sharing platform) in the prosumer market that can measure the aggregate response of the prosumer community and broadcast a feedback signal in the prosumer market at each time instant t_k, for both consumption and production type. Specifically, the sharing platform updates the feedback signal $\Omega_x(k)$, as follows:

$$\Omega_x(k+1) \triangleq \Omega_x(k) - \tau_x\Big(\sum_{i=1}^{N} x_i(k) - C_x\Big), \quad \text{for all } k, \tag{21}$$

with $\Omega_y(k)$ updated analogously. After receiving a feedback signal prosumer i's algorithm responds in a probabilistic manner. The probability that prosumer i responds to the feedback signal is given by:

$$\sigma_{i,x}(k) \triangleq \Omega_x(k)\frac{\nabla_x g_i\big(\overline{x}_i(k), \overline{y}_i(k)\big)}{\overline{x}_i(k)}, \quad \text{for all } i \text{ and } k. \tag{22}$$

Here, $\sigma_{i,x}(k)$ denotes the probability of prosumer i, responding to a demand for consumption of the resource, at time instant t_k; similarly, $\sigma_{i,y}(k)$ denotes the probability of prosumer i, responding for production of the resource at time instant t_k, for all k. As in the consumption case, the cost function g_i is chosen as increasing function in each variable, to keep the probabilities $\sigma_{i,x}(k)$ and $\sigma_{i,y}(k)$ in the valid range. However, the definition of $\sigma_{i,x}(k)$ and $\sigma_{i,y}(k)$ is slightly different from the consumption case, described previously. Now, prosumer i updates its consumption and production of the resource at each time instant in the following way:

$$x_i(k+1) = \begin{cases} 1 & \text{with probability } \sigma_{i,x}(k); \\ 0 & \text{with probability } 1 - \sigma_{i,x}(k), \quad \text{and,} \end{cases}$$

$$y_i(k+1) = \begin{cases} 1 & \text{with probability } \sigma_{i,y}(k); \\ 0 & \text{with probability } 1 - \sigma_{i,y}(k), \end{cases}$$

for all i and k. Empirically, we observe that the time-averaged consumption $\overline{x}_i(k)$ converges to the optimal consumption value x_i^*, and similarly, the time-averaged production $\overline{y}_i(k)$ converges to the optimal production value y_i^* asymptotically, for all prosumers in the community. We describe the algorithm of the sharing platform in Algorithm 1 and the algorithm of prosumers of the community in Algorithm 2. We make the following remarks.

Remark 3 (Communication overhead and privacy) There is no explicit communication between prosumers. Thus, the algorithm is low cost in terms of communication and is private.

Remark 4 (Probability bounds) The gain parameters τ_x, τ_y are small constants chosen to ensure $\sigma_{i,x}(k)$ and $\sigma_{i,y}(k)$ are the probabilities; namely, are in [0,1], for all i and k.

Remark 5 (Consensus of partial derivatives) The well-posedness of our algorithm follows from the assumption of strict convexity of $g_i(\cdot)$, and that the constraint sets are closed and bounded; namely, that there exists unique optimal solutions to Problem 1. To show that the long-term average values converge to the optimal values, we use the consensus of (partial) derivatives of the cost functions of prosumers as described previously. That is:

$$\lim_{k\to\infty} \nabla_x g_i(\overline{x}_i(k), \overline{y}_i(k)) = \lim_{k\to\infty} \nabla_x g_j(\overline{x}_j(k), \overline{y}_j(k)),$$

and similarly,

$$\lim_{k\to\infty} \nabla_y g_i(\overline{x}_i(k), \overline{y}_i(k)) = \lim_{k\to\infty} \nabla_y g_j(\overline{x}_j(k), \overline{y}_j(k)),$$

for all $i, j \in \{1, 2, \ldots, N\}$.

Algorithm 1: Algorithm of control unit (sharing platform).

---
1 Input: $C_x, C_y, \tau_x, \tau_y, x_i(k), y_i(k)$, for $k = 0, 1, 2, \ldots$ and $i = 1, 2, \ldots, N$.
2 Output: $\Omega_x(k+1), \Omega_y(k+1)$, for $k = 0, 1, 2, \ldots$
3 Initialization: $\Omega_x(0) \leftarrow 0.06$ and $\Omega_y(0) \leftarrow 0.06$[1],
4 foreach $k = 0, 1, 2, \ldots$ **do**
5   calculate $\Omega_x(k+1)$ and $\Omega_y(k+1)$ as follows and broadcast them in the prosumer community;

$$\Omega_x(k+1) \leftarrow \Omega_x(k) - \tau_x\Big(\sum_{i=1}^{N} x_i(k) - C_x\Big), \quad \text{and,}$$

$$\Omega_y(k+1) \leftarrow \Omega_y(k) - \tau_y\Big(\sum_{i=1}^{N} y_i(k) - C_y\Big).$$

6 end

---

5 Use Cases

We now describe two use cases for community-based prosumer market. The first is a transportation example and concerns a car sharing prosumer market. The second example is from the energy sector and considers a prosumer market, where produced and consumed energy is coupled through storage constraints.

---

[1] We initialize $\Omega_x(0)$ and $\Omega_y(0)$ with positive real numbers.

Algorithm 2: Algorithm of prosumer i.

---

1 Input: $\Omega_x(k), \Omega_y(k)$, for $k = 0, 1, 2, \dots$
2 Output: $\overline{x}_i(k+1), \overline{y}_i(k+1)$, for $k = 0, 1, 2, \dots$
3 Initialization: $x_i(0), y_i(0) \leftarrow 1$ and $\overline{x}_i(0) \leftarrow x_i(0)$ and $\overline{y}_i(0) \leftarrow y_i(0)$.
4 **while** *prosumer i is active at $k = 0, 1, 2, \dots$* **do**
5

$$\sigma_{i,x}(k) \leftarrow \Omega_x(k) \frac{\nabla_x g_i\big(\overline{x}_i(k), \overline{y}_i(k)\big)}{\overline{x}_i(k)};$$

$$\sigma_{i,y}(k) \leftarrow \Omega_y(k) \frac{\nabla_y g_i\big(\overline{x}_i(k), \overline{y}_i(k)\big)}{\overline{y}_i(k)};$$

calculate outcome of the random variables;

$$x_i(k+1) \leftarrow \begin{cases} 1 & \text{w. p. } \sigma_{i,x}(k) \\ 0 & \text{w. p. } 1 - \sigma_{i,x}(k); \end{cases}$$

$$y_i(k+1) \leftarrow \begin{cases} 1 & \text{w. p. } \sigma_{i,y}(k) \\ 0 & \text{w. p. } 1 - \sigma_{i,y}(k); \end{cases}$$

update $\overline{x}_i(k+1)$ and $\overline{y}_i(k+1)$ as follows;

$$\overline{x}_i(k+1) \leftarrow \frac{k+1}{k+2} \overline{x}_i(k) + \frac{1}{k+2} x_i(k+1);$$

$$\overline{y}_i(k+1) \leftarrow \frac{k+1}{k+2} \overline{y}_i(k) + \frac{1}{k+2} y_i(k+1);$$

6 **end**

5.1 Community-based Car Sharing

In this use case, we consider a community of N households (prosumers) with several cars, not all of which are required each day. We assume that cars are pooled and shared amongst community members to multiplex and monetize the excess capacity, but that the average aggregate daily community demand for cars is known. We let T_i denote the average number of cars desired to be used by each household i, for all i. Suppose that C_x cars are required within the community each day, and an excess of C_y cars are made available to an external community each day. For simplicity, we assume that C_x and C_y are fixed. Notice that a household is a prosumer in the sense that it requires cars for its transportation needs and supplies excess car-days to an external community. For each prosumer i, let $x_i(k) \in \{0, 1\}$ denote that prosumer i requires a car at day k or not, and let $y_i(k) \in \{0, 1\}$ denote that prosumer i supplies a car to an external community at day k or not. Thus, we assume that aggregated over the entire community, the demand for *shared cars* on a given day k is:

$$\sum_{i=1}^{N} x_i(k) = C_x, \tag{23}$$

leaving the *excess* capacity so that C_y cars can be supplied to an external community:

$$\sum_{i=1}^{N} y_i(k) = C_y. \tag{24}$$

Thus, $\sum_{\ell=0}^{k} x_i(\ell)$ is the number of days a car was required by prosumer i over k days, and $\sum_{\ell=0}^{k} y_i(\ell)$ is the number of days that the same prosumer made a car available to an external community. Thus, $\sum_{\ell=0}^{k} \big(x_i(\ell) + y_i(\ell)\big)$ is the total number of days that a car was used as a result of prosumer i. Over some days, prosumer i will require that the time-averaged of this number be equal to the desired value of utilization of cars, T_i. For example, if a prosumer has two cars, thus, over seven days period (a week) it has 14 car-days. Now, suppose that the prosumer only needs access to 10 car-days over a week. Then, this prosumer might choose $\sum_{\ell=0}^{6} x_i(\ell)$ and $\sum_{\ell=0}^{6} y_i(\ell)$, such that $\sum_{\ell=0}^{6} \big(x_i(\ell) + y_i(\ell)\big) = 12$. In this case, the prosumer sells access to two excess car-days over the week (two car-days rather than the maximum of four to provide some margin in case that a car is required for personal use more than expected). Let $\overline{x}_i(k)$ denote the average number of days prosumer i requires a car over k days, and let $\overline{y}_i(k)$ denote the average number of days prosumer i makes a car available to an external community over k days, for all i and k. Thus, over a long period, this constraint can be scaled by the number of days to yield:

$$\overline{x}_i(k) + \overline{y}_i(k) \approx T_i, \tag{25}$$

with the cost of not achieving this goal captured by a penalty function $g_i(\overline{x}_i + \overline{y}_i - T_i)$. For sufficiently large k, we might also require that $\overline{x}_i(k) \approx \alpha_i T_i$ and $\overline{y}_i(k) \approx (1 - \alpha_i)T_i$ with $\alpha_i \in [0, 1]$, for all i. This latter constraint can be formulated in terms of a cost via a penalty function. For example, in residential areas in Ireland; two car households are common, meaning that households can in principle both consume and produce cars simultaneously, hence act as prosumers. However, this may not always be possible, and prosumers may be required to make alternative arrangements or pay penalties, should they not be able to meet contractual demands. To formulate the cost function, we associate costs $h_i : (0, 1] \to \mathbb{R}_+, x_i \mapsto h_i(x_i)$ to the deviation from $\alpha_i T_i$ and $l_i : (0, 1] \to \mathbb{R}_+, y_i \mapsto l_i(y_i)$ to the deviation from $(1 - \alpha_i)T_i$, for all i. Then, the aim of the sharing platform is to minimize:

$$\sum_{i=1}^{N} \big(g_i(\overline{x}_i + \overline{y}_i - T_i) + h_i(\overline{x}_i - \alpha_i T_i) + l_i(\overline{y}_i - (1 - \alpha_i)T_i)\big), \tag{26}$$

subject to the additional constraints listed in Problem 1.

5.2 Collaborative Energy Storage

In this use case, we again assume that there are N households in a community participating in a prosumer market, with each household connected to the grid, and also has installed solar panels. Every household has batteries to store the energy either from the solar panel or the grid. The households act as prosumers in the sense that they can consume stored energy as well as sell excess energy for monetary benefits. Let $x_i(k) \in \{0,1\}$ denote that household i consumes stored energy at day k or not, and let $y_i(k) \in \{0,1\}$ denote that household i sells stored energy to an external community at day k or not, for all i. Let $\overline{x}_i(k)$ denote the time-averaged amount of stored energy consumed by household i over k days, and let $\overline{y}_i(k)$ denote the time-averaged amount of stored energy sold by household i to an external community over k days, for all i. Now, we assume that $\sum_{i=1}^{N} x_i(k) = C_x$, be the aggregated consumption of stored energy over the entire community on a given day k; whereas, the excess energy C_y is sold to an external community, with $\sum_{i=1}^{N} y_i(k) = C_y$. Furthermore, we assume that each household may have a constraint on the amount of energy stored in order to realize the above strategy. As in the previous use case, this *soft* constraint is captured as follows. Let T_i be the desired amount of energy stored by household i. Then, on average, over a sufficiently large given period k, household i may expect to store the following desired amount of energy temporarily:

$$\overline{x}_i(k) + \overline{y}_i(k) \approx T_i, \tag{27}$$

with the deviation from this goal captured by a penalty function $g_i(\overline{x}_i + \overline{y}_i - T_i)$. We might also require that roughly speaking, a certain amount of storage is reserved for consumption and a certain amount to sell (production). Hence, again, the objective of the sharing platform is to minimize:

$$\sum_{i=1}^{N} \big(g_i(\overline{x}_i + \overline{y}_i - T_i) + h_i(\overline{x}_i - \alpha_i T_i) + l_i(\overline{y}_i - (1 - \alpha_i)T_i) \big),$$

subject to the constraints listed in Problem 1. Notice that the definition of $h_i(\cdot)$ and $l_i(\cdot)$ is same as the previous use case.

6 Simulation Results

In this section, we present simulation results for 99 prosumers participating in a prosumer market. We assume that these $N = 99$ in the prosumer market are grouped into two prosumer communities. Prosumers 1 to 50 are grouped in Community 1, and Prosumers 51 to 99 are grouped in Community 2. Additionally, each community has a cost function type, and prosumers of a particular community use the cost function type of that community, but with randomized parameter values. Recall that prosumers can produce the resource as well as consume it. Let time instants $t_0 < t_1 < t_2 < \ldots$, represent days, and let

the capacity constraints be $C_x = 90$ and $C_y = 80$. Furthermore, let the cost factors $a_i \in \{1, 2, \ldots, 10\}$ and $b_i \in \{1, 2, \ldots, 15\}$ be drawn from uniformly distributed random variables. Additionally, let $\delta = 11.75$ and $\gamma = 11.65$. Notice that the values of δ and γ are chosen in such a way that the cost function $g_i(\cdot)$ is increasing in each variable. Recall that this is done to keep the probabilities $\sigma_{i,x}$ and $\sigma_{i,y}$ in $[0, 1]$. The cost functions are presented as follows:

$$
g_i(\overline{x}_i(k), \overline{y}_i(k)) =
\begin{cases}
\delta\big(\overline{x}_i(k) + \overline{y}_i(k)\big) + \frac{1}{4}a_i\big(\overline{x}_i(k) + \overline{y}_i(k) - T_1\big)^2 + \\
\frac{1}{2}\big(\overline{x}_i(k) - \frac{1}{2}T_1\big)^2 + \frac{1}{2}\big(\overline{y}_i(k) - \frac{1}{2}T_1\big)^2 \qquad \text{Community 1,} \\
\\
\gamma\big(\overline{x}_i(k) + \overline{y}_i(k)\big) + \frac{1}{8}a_i\big(\overline{x}_i(k) + \overline{y}_i(k) - T_2\big)^2 + \\
\frac{5}{4}b_i\big(\overline{x}_i(k) + \overline{y}_i(k) - T_2\big)^4 + \frac{1}{2}\big(\overline{x}_i(k) - \frac{1}{2}T_2\big)^2 + \\
\frac{1}{2}\big(\overline{y}_i(k) - \frac{1}{2}T_2\big)^2 \qquad \text{Community 2.}
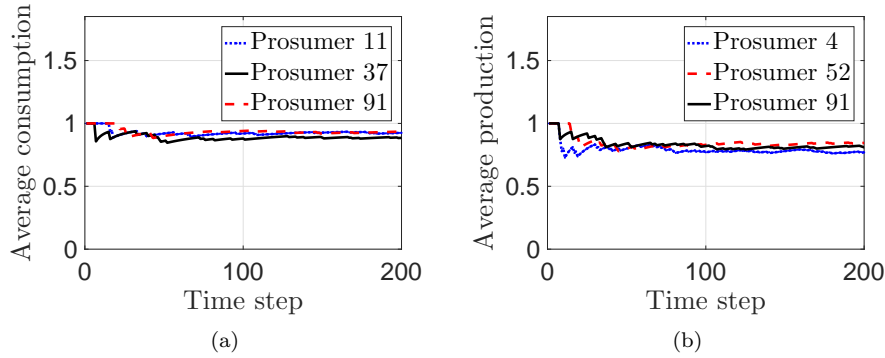\end{cases}
$$

Fig. 4: (a) Evolution of time-averaged consumption of the resource, and (b) evolution of time-averaged production of the resource.

Now, we present the simulation results and show the convergence of the long-term average of prosumption by each prosumer in Figure 4. Figure 4(a) shows the time-averaged consumption and Figure 4(b) shows the time-averaged production over 200 days. In the context of car sharing, these are the average of cars used by a prosumer and the average of cars shared with another person by the same prosumer over 200 days, respectively. Figure 5 illustrates the average prosumption on the 200$^{\text{th}}$ day by every prosumer of a particular community. Furthermore, the absolute difference between the desired value of utilization T_i of cars and the actual utilization $\overline{x}_i(k) + \overline{y}_i(k)$ by prosumer i for a certain period is shown in Figure 6. Figure 6(a) illustrates the evolution of the absolute difference between T_i and $\overline{x}_i(k) + \overline{y}_i(k)$ for individual prosumers. Here, we observe that gradually the difference comes closer to zero. Additionally, in Figure 6(b) and 6(c), we observe that the absolute difference between the quantities is close to zero for most of the prosumers.
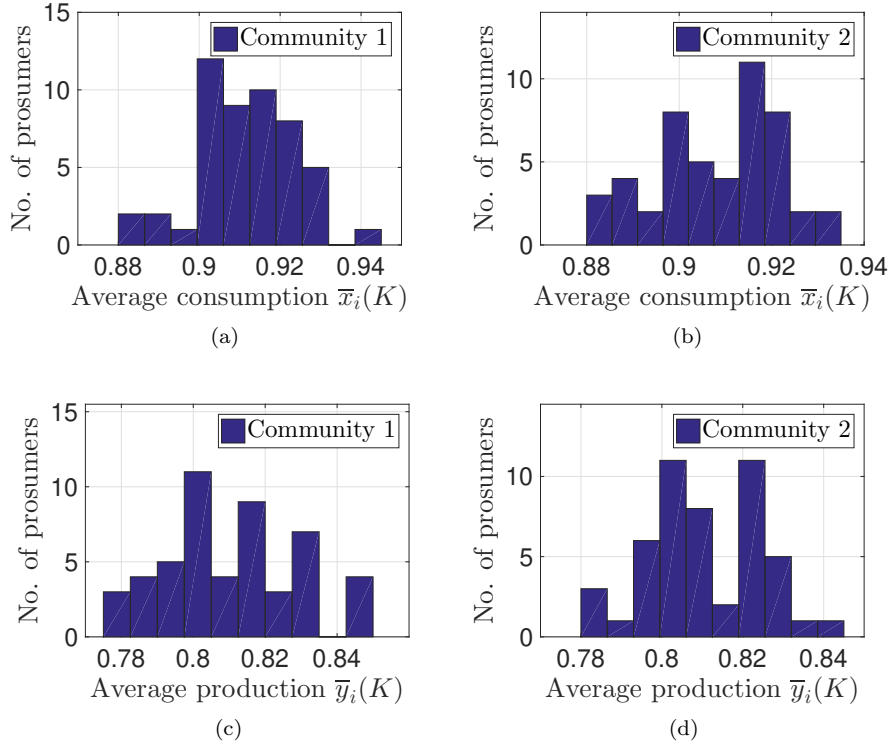
Fig. 5: At time index $K = 200$—(a) average consumption $\overline{x}_i(K)$ by prosumers of Community 1, (b) average consumption $\overline{x}_i(K)$ by prosumers of Community 2, (c) average production $\overline{y}_i(K)$ by prosumers of Community 1, and (d) average production $\overline{y}_i(K)$ by prosumers of Community 2.

Now, we analyze the derivatives of the cost functions and see, whether they gather close to each other to make consensus over time or not. Recall that the derivative of the cost function with respect to consumption is $\nabla_x g_i(.)$ and with respect to production is $\nabla_y g_i(.)$, that are shown in Figure 7 for a single simulation. We plot the shaded errorbars as depicted in Figure 7(a) and 7(b). It is observed that in both the cases the derivatives gather close to each other over time. Therefore, we say that the derivatives make consensus asymptotically in their respective prosumer communities, which is a necessary and sufficient condition for optimality as described in Subsection 4.1. Notice that because both the derivatives (with respect to consumption and production) are same, therefore, we illustrate here just one of them. We clarify here that because of the chosen initial values, the probability $\sigma_{i,x}(k)$ may overshoot at the start of the algorithm; to keep it in the valid range, we use $\min\left\{1, \Omega_x(k)\frac{\nabla_x g_i\left(\overline{x}_i(k),\overline{y}_i(k)\right)}{\overline{x}_i(k)}\right\}$.

(a)

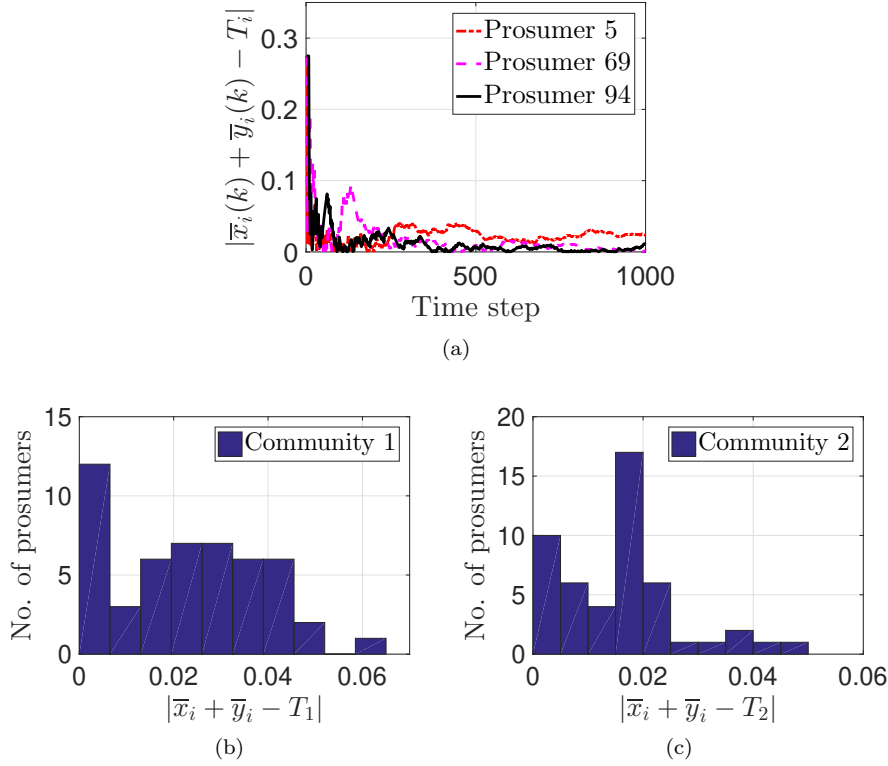(b)   (c)

Fig. 6: (a) Evolution of absolute difference between desired value of utilization T_i and actual utilization of the resource $\overline{x}_i(k) + \overline{y}_i(k)$ of individual prosumers, (b) absolute difference between desired value of utilization and actual utilization $|\overline{x}_i(K) + \overline{y}_i(K) - T_1|$ of prosumers of Community 1, here $T_1 = 1.74$, and (c) absolute difference between desired value of utilization and actual utilization $|\overline{x}_i(K) + \overline{y}_i(K) - T_2|$ of prosumers of Community 2, here $T_2 = 1.725$ and time index $K = 200$.

Similar step is used to keep $\sigma_{i,y}(k)$ in the valid probability range.

Now, we analyze the aggregate consumption $\sum_{i=1}^{N} x_i(k)$ and production $\sum_{i=1}^{N} y_i(k)$ by the prosumer communities. The aggregate consumption $\sum_{i=1}^{N} x_i(k)$ is presented in Figure 8(a) for last 40 time instants (days), and similarly, the aggregate production by the communities $\sum_{i=1}^{N} y_i(k)$ is shown in Figure 8(b). Notice that the aggregate prosumption is close to the respective capacity constraints C_x and C_y; overshoots and undershoots are due to the assumption of soft constraints, as described previously. Additionally, Figure 8(c) shows the time-averaged consumption $\sum_{i=1}^{N} \overline{x}_i(k)$ by all prosumers in the prosumer market until 1000 time instants, and similarly, the time-averaged production
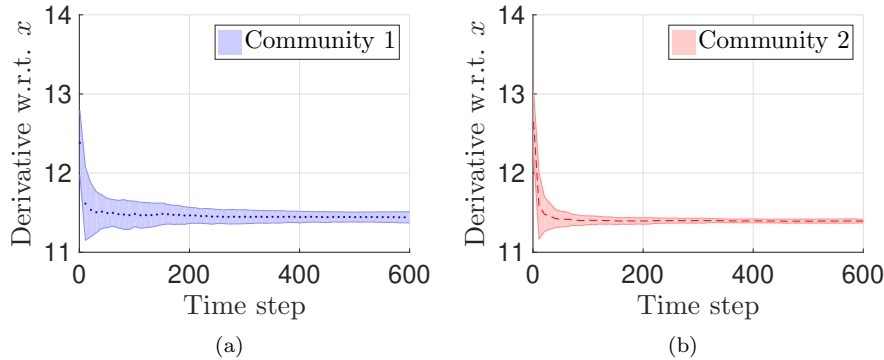
Fig. 7: (a) Evolution of derivatives of $g_i(.)$ w. r. t. x for prosumers of Community 1, and (b) evolution of derivatives of $g_i(.)$ w. r. t. x for prosumers of Community 2.

$\sum_{i=1}^{N} \overline{y}_i(k)$ by all prosumers in the prosumer market for the same period, these averages are approximately equal to the respective capacities, satisfying the capacity constraints of Problem 1. In addition to the above results, we observe in Figure 9(a) and 9(b) that most of the time the aggregate consumption $\sum_{i=1}^{N} x_i(k)$ and the aggregate production $\sum_{i=1}^{N} y_i(k)$ are close to their respective capacities C_x and C_y. Furthermore, the convergence of feedback signals Ω_x and Ω_y are shown in Figure 9(c).

7 Conclusion and Future Directions

We proposed distributed control algorithms to solve regulation problems with optimality constraints for community-based prosumer market. The algorithm is based on ideas from stochastic approximation but formulated in a control theoretic setting. The algorithm reaches optimality asymptotically, while simultaneously regulating instantaneous contract constraints. To do so, the algorithm does not require communication between prosumers, but little communication with the sharing platform. Additionally, the algorithm is light and is suitable to implement in an Internet-of-Things (IoT) context with minimal demands on infrastructure. Two applications are described and simulation results presented to demonstrate the efficacy of the algorithms. Future work will explore the theoretical aspects of the algorithm (convergence properties), new applications and use cases, and the development of policies to reach more complicated equilibria.
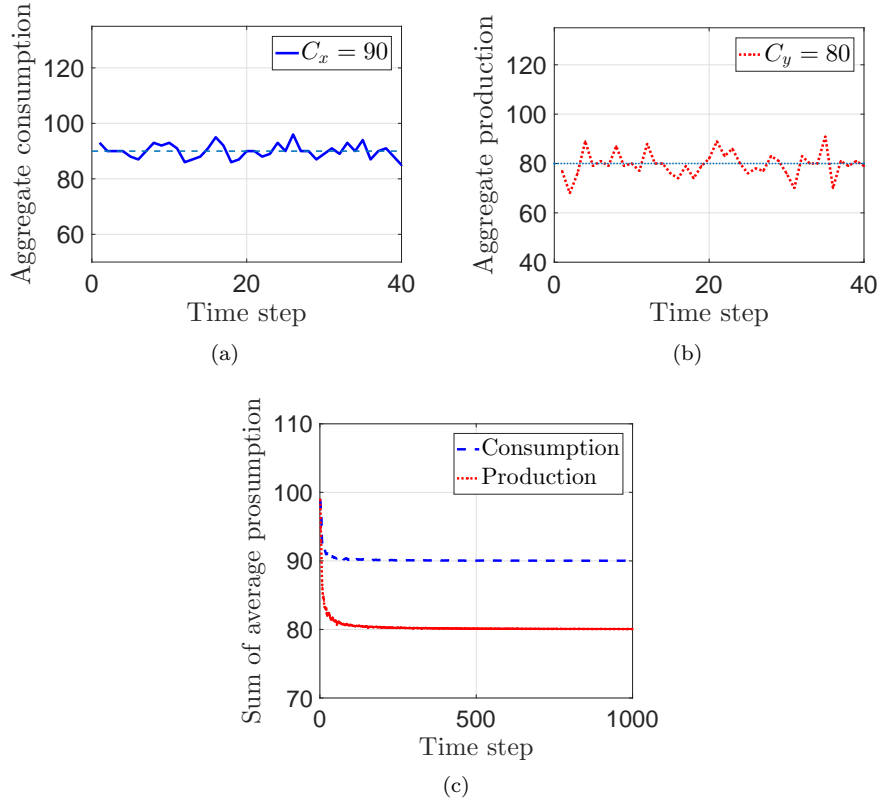
Fig. 8: Aggregate prosumption for last 40 time instants—(a) aggregate consumption $\sum_{i=1}^{N} x_i(k)$, (b) aggregate production $\sum_{i=1}^{N} y_i(k)$, and (c) evolution of sum of time-averaged prosumption.

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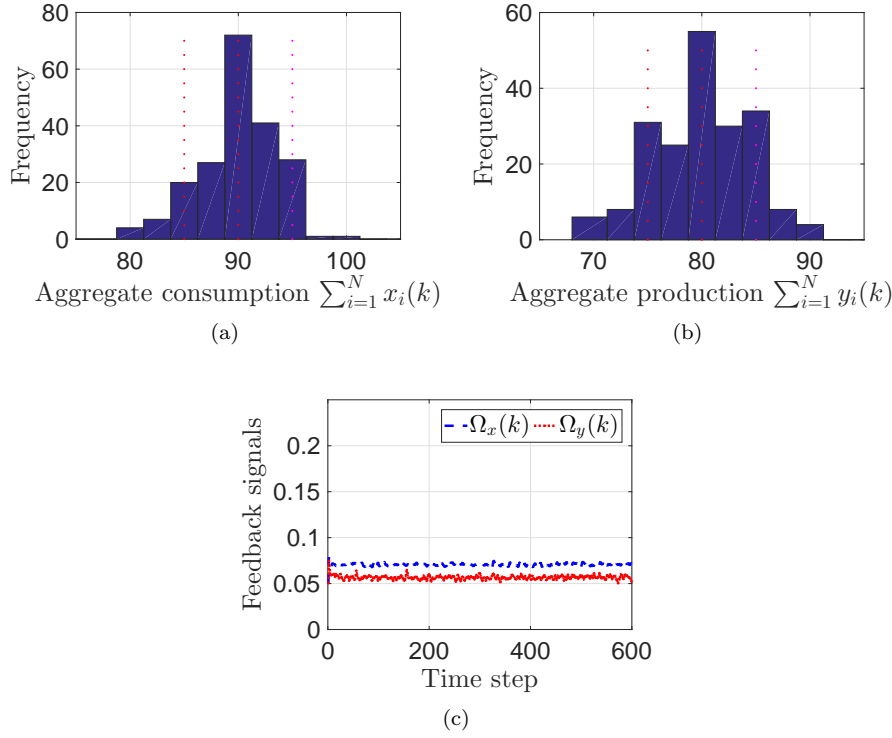
Fig. 9: Frequency of prosumption—(a) frequency of aggregate consumption, (b) frequency of aggregate production for last 200 time instants, and (c) evolution of feedback signals $\Omega_x(k)$ and $\Omega_y(k)$.

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